Preface

In chapter '3 Hull-propeller interactions' of the above mentioned paper under '3.3 Merits of different propulsors' it has been stated:

"Triggered by his paper on the rational evaluation of ducted propulsors in open water (2007/2009) and by the consideration of Sistemar's CLT Propellers according to the design of Gomez (Gennaro, 2008) the author, during the preparation of this manuscript, felt the necessity to revise his 'instinctive beliefs' concerning the objective comparison of the merits of different propulsor configurations.

In case of propulsors in open water the ratio of the actual propulsive efficiency and of the ideal or jet efficiency, the hydraulic or pump efficiency, has been proposed as objective measure of merit. But the author has 'never' appropriately questioned and scrutinized the definition of the jet efficiency, also fundamental for the determination of thrust deduction and wake fractions.

In case of 'open' propellers the jet efficiency has naively been based on the thrust loading of the propeller in the same way as in case of the ideal propulsors producing ideal jets. As long as only open propellers are compared, this procedure may be sufficient, but in general it is felt to be inadequate in principle.

In terms of pump theory only propellers with the same power and the same flow rate can be compared. The same condition has been observed earlier to define equivalent propulsors in the theory of hull-propeller interactions and to arrive, among others, at the thrust deduction theorem.

Accordingly a procedure permitting an objective evaluation of the merits of different propulsor configurations has been conceived and will be tested numerically before being..
In general only the area and the thrust of the 'rotor' are available as data and accordingly the rotor has to be 'calibrated' as flow-meter. This can be achieved ideally by assuming constant axial velocity before and behind the rotor and by assuming a potential vortex leaving the rotor and hence the corresponding tangential velocities behind the rotor.

In the following the hydraulic efficiencies of a propeller model in open water are determined based on thrust, momentum flow, and based on power, energy flow, i.e. considering the propeller as ordinary propeller without stator and as rotor of a propulsor with stator, respectively.

**Preliminaries**

Mathcad permits to handle physical quantities, but all data are being used without their SI units in view of further use in mathematical subroutines, which by definition cannot handle arguments with units.

**Units**

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbol</th>
<th>Unit</th>
<th>Conversion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Force</td>
<td>N</td>
<td>newton</td>
<td>kp = g · N</td>
</tr>
<tr>
<td>Torque</td>
<td>Nm</td>
<td>newton·m</td>
<td></td>
</tr>
<tr>
<td>Power</td>
<td>W</td>
<td>watt</td>
<td></td>
</tr>
</tbody>
</table>

**Constants**

<table>
<thead>
<tr>
<th>Field</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gravity field</td>
<td>g</td>
<td>9.81 m·sec⁻²</td>
</tr>
<tr>
<td>Temperature</td>
<td>g</td>
<td>g = m⁻¹·sec²</td>
</tr>
</tbody>
</table>

**Routines**

**Left inverse**

\[
\text{LeftInv}(A) := \begin{cases}
    r \leftarrow \text{rows}(A) \\
    c \leftarrow \text{cols}(A) \\
    s \leftarrow \text{svds}(A) \\
    \text{for } i \in 0..c - 1 \\
    ISV_{i,i} \leftarrow \left(s_i\right)^{-1} \\
    UV \leftarrow \text{svd}(A) \\
    U \leftarrow \text{submatrix}(UV, 0, r - 1, 0, c - 1) \\
    V \leftarrow \text{submatrix}(UV, r, r + c - 1, 0, c - 1) \\
    A_{\text{inv.left}} \leftarrow V \cdot ISV \cdot U^T \\
    A_{\text{inv.left}} 
\end{cases}
\]
VWS Propeller Model 1340

Basic data

CP propeller, right handed

Diameter of rotor alias propeller

\[ D_R := 0.195 \cdot \text{m} \]

\[ R_R := \frac{D_R}{2} \]

\[ R_R = 0.0975 \]

Diameter of hub, crudely estimated

\[ D_{HR} := 0.0195 \cdot \text{m} \]

\[ R_{HR} := \frac{D_{HR}}{2} \]

\[ R_{HR} = 0.0098 \]

\[ \delta := \frac{D_{HR}}{D_R} \]

\[ \delta = 0.1000 \]

\[ \alpha := (1 - \delta^2) \]

\[ \alpha = 0.9900 \]

Flow area

\[ A_R := \pi \cdot R_R^2 \cdot \alpha \]

\[ A_R = 0.0296 \]

Pitch ratio, design

\[ P_{D,\text{des}} := 0.825 \]

Pitch ratio, actual

\[ P_{D,\text{act}} := 0.813 \]

Number of blades

\[ Z := 4 \]

Rate of revolutions at open water test

\[ n_{\text{open}} := 12 \cdot \text{Hz} \]

Density of tank water

\[ \rho := 1.00 \cdot 10^3 \cdot \text{kg} \cdot \text{m}^{-3} \]

\[ \rho := \rho \cdot \text{kg}^{-1} \cdot \text{m}^3 \]
Open water data

\[
\begin{bmatrix}
0.35 & 48.0 & 63.5 \\
0.40 & 43.0 & 59.5 \\
0.45 & 38.0 & 53.0 \\
0.50 & 33.0 & 48.0 \\
0.55 & 28.0 & 43.0 \\
0.60 & 22.5 & 37.5 \\
0.65 & 17.5 & 32.0 \\
\end{bmatrix}
\]

KT and 10 KQ values read in mm from Fig. 0.2 in VWS Bericht Nr.1126/88

scale := 200

\[
\begin{align*}
\text{Data}_\text{prop} := \\
J_{\text{P.raw}} & := \text{Data}_\text{prop}^{<0>} \\
K_{\text{T.raw}} & := \frac{\text{Data}_\text{prop}^{<1>}}{\text{scale}} \\
K_{\text{Q.raw}} & := \frac{\text{Data}_\text{prop}^{<2>}}{\text{scale}} \\
K_{\text{P.raw}} & := \frac{2\cdot\pi}{10} \cdot K_{\text{Q.raw}}
\end{align*}
\]

Interpolation

\[
j := 0..1
\]

\[
k := 0..\text{last}(J_{\text{P.raw}})
\]

\[
\begin{align*}
A_{\text{JP.raw}^{k,j}} & := \langle J_{\text{P.raw}}^{k} \rangle^{j} \\
X_{\text{KT.open}} & := \text{LeftInv}(A_{\text{JP.raw}}) \cdot K_{\text{T.raw}} \\
X_{\text{KP.open}} & := \text{LeftInv}(A_{\text{JP.raw}}) \cdot K_{\text{P.raw}} \\
K_{\text{TP}} & := A_{\text{JP.raw}} \cdot X_{\text{KT.open}} \\
K_{\text{PP}} & := A_{\text{JP.raw}} \cdot X_{\text{KP.open}}
\end{align*}
\]

Thrust and power ratios as functions of propeller open water advance ratio

\[
k_{\text{T.open}}^{(j \cdot P)} := \sum_{j} X_{\text{KT.open}^{j}}^{(j \cdot P)}
\]

\[
k_{\text{P.open}}^{(j \cdot P)} := \sum_{j} X_{\text{KP.open}^{j}}^{(j \cdot P)}
\]

\[
c_{\text{T.open}}^{(j \cdot P)} := \frac{8}{\pi \cdot j \cdot P^{2} \cdot \alpha} \cdot k_{\text{T.open}}^{(j \cdot P)}
\]
Values interpolated

\[ i := 0..45 \]

\[ J_{P, \text{open}}^i := 0.0001 + i \cdot 0.02 \]

\[ K_{T, \text{open}}^i := k_{T, \text{open}} \left( J_{P, \text{open}}^i \right) \]

Thrust ratios

\[ K_{T, \text{raw}} \]

\[ K_{T, \text{open}} \]

Power ratios

\[ K_{P, \text{raw}} \]

\[ K_{P, \text{open}} \]
**Propulsor efficiencies: without stator**

**Propulsive efficiency**

\[ \eta_{TP, open} := \frac{K_{T, open} \cdot J_{P, open}}{K_{P, open}} \]

\[ \eta_{TP, open} \]

**Jet efficiency, nominal**

\[ C_{T, open} := c_{T, open} \left( J_{P, open} \right) \]

\[ \eta_{TJ, T, open} := \frac{2}{1 + \sqrt{1 + C_{T, open}}} \]

**Jet efficiency, nominal**
Hydraulic efficiency, nominal

\[ \eta_{JP.T.open_i} := \frac{\eta_{TP.open_i}}{\eta_{TJ.T.open_i}} \]
**Propulsor efficiencies: with stator**

As has been stated in the quotation found in the Preface the thrust at rotors of propulsors is not a measure of the energy fed into the fluids due to the axial vortices or the tangential velocities behind the rotors. Thus the simple minded evaluation of the merits of propulsors based on thrust is not satisfactory if stators are available.

The following is the first attempt to provide a more adequate evaluation and compare it with the evaluation of open propellers.

As has been stated in the paper quoted the underlying assumptions are uniform axial flow velocity before and behind the rotor and a potential vortex behind the rotor.

According to the model adopted!

\[
a(j_p) := \frac{\pi^2 (1 - \delta^2)}{\ln \left( \frac{1}{\delta} \right)}
\]

\[
c_E.\text{open}(j_p) := a(j_p) \cdot \left( 1 - \sqrt{1 - \frac{2 \cdot c_T.\text{open}(j_p)}{a(j_p)}} \right)
\]

Thus in the limit of small propeller loading etc as it 'must' be:

\[
c_E.\text{open}\text{.limit}(j_p) := c_T.\text{open}(j_p)
\]

\[
C_{E.\text{open}_i} := c_E.\text{open}(j_{P.\text{open}_i})
\]

\[
\eta_TJ.E.\text{open}_i := \frac{2}{1 + \sqrt{1 + C_{E.\text{open}_i}}}
\]

Today's scrutiny has revealed a mistake in the earlier derivation of the parameter. The correct values are by a factor 2 larger.
Some conclusions

The differences in the results in the case considered are due to the reduction in thrust due to the axial vorticity in the jet.

Though the differences in jet efficiencies are quite small the result confirms the 'suspicion' that the values of the hydraulic efficiency based on thrust measured at the rotors are not adequate measures of performance in general. While in the range of moderate loading the difference is small, at higher loading the differences between thrust and power based hydraulic efficiencies are considerable, though not as large as expected and obtained obtained earlier using the too small values of the parameter.

This fact has always been known and has lead to the various proposals for pre- and post-swirl configurations using 'existing' surfaces in order to avoid additional frictional losses. Most successful have been Star-Contra-Configurations before World War II, when already a considerable percentage of the world fleet had been equipped with such configurations. Horn in a short paper noted that in case of highly loaded ducted propulsors a stator would always be of advantage, maybe just twisting the rudders.

The procedure to compute the power or energy loading outlined applies not only in case of open propellers but in case of any other propulsor configuration, ducted propulsors in particular, where measurements usually can be performed only at the rotors.

The problem is that often the value of the total thrust necessary to determine the hydraulic efficiency cannot be measured. How to solve this problem has been explained elsewhere.
According to the analysis the hydraulic efficiency based on thrust loading is appropriate for open propellers without stator, while the hydraulic efficiency based on power or energy loading of the rotor is appropriate for propellers with pre- and post-swirl devises and ducted propulsors with stators.