Preface

The basis of the 'rational' full scale ship powering performance prediction based on model tests to be developed are 'rational' procedures of model testing and of evaluating the model powering performance. Such procedures based on quasi-steady propulsion tests with ship models have been described and demonstrated to be feasible using VWS ship model 2491.0 and propeller model 1340 in the final report VWS Bericht Nr. 1105/88 on the project and in the preliminary report:


The essential parts of this report, including body plan and the contours of stem and stern, will constitute the first appendix of the paper. They are to be found on the website of the author as well. Warning: the file is large, nearly 1 MB!
The subject of this document is to re-re-evaluate the sample model data in that report based on the insight and experience gained over the past fifteen years and during the months of April and May 2008. In particular the local axioms or constitutive laws of wake and thrust deduction have been scrutinised again, triggered by questions of Dr.-Ing. habil. Klaus Wagner of Rostock.

The following exercise shows that nearly all the unsolved problems have finally been solved, the solution of the energy wake problem still open. The test case shows that the model powering performance in a wide range of hull advance ratios can be derived from the data of only one run down the model basin, may be using freely moving models, not requiring a towing carriage. Evidently the same technique can be applied on full scale. Thus in both cases a dramatic gain in reliability and cost effectivity can be obtained.

The Mathcad document and the data file will be made available on request. Despite extreme care in every detail the evaluation may still contain inconsistencies and/or errors. The author will be most grateful for any communication, not only concerning such mistakes, but maybe concerning lack of clarity in the exposition, questions arising and experience gained in applications.

'Unneccesary' to mention that in routine applications the programming will be quite different, typically in terms of subroutines, which have been used only occasionally in this document. But in view of the sensitivity of the problem at hand colleagues are warned: there will be 'no plug and play' program. In any case careful scrutiny of data and intermediate results is absolutely mandatory.

And to repeat: The method proposed offers dramatic technological and commercial advantages. No propeller open water and hull towing tests are necessary and the extremely short propulsion tests provide a wealth of consistent data and results.

Preliminaries

Mathcad permits to handle physical quantities, but all data are being used without their SI units in view of further use in mathematical subroutines, which by definition cannot handle arguments with units.

### Constants

- **Gravity field**
  
  \( g := 9.81 \text{ m sec}^{-2} \)

- **Units**

  - **Force**
    
    \( N := \text{newton} \)
  
  - **Torque**
    
    \( Nm := \text{newton\cdot m} \)
  
  - **Power**
    
    \( W := \text{watt} \)
Routines

Left inverse

\[
\text{LeftInv}(A) := \begin{align*}
& \text{r} \leftarrow \text{rows}(A) \\
& \text{c} \leftarrow \text{cols}(A) \\
& \text{s} \leftarrow \text{svds}(A) \\
& \text{for } i \in 0..c - 1 \\
& \quad \text{ISV}_{i,i} \leftarrow \left(s_i\right)^{-1} \\
& \text{UV} \leftarrow \text{svd}(A) \\
& \quad \text{U} \leftarrow \text{submatrix}(UV, 0, r - 1, 0, c - 1) \\
& \quad \text{V} \leftarrow \text{submatrix}(UV, r, r + c - 1, 0, c - 1) \\
& \text{A} \leftarrow \text{inv.left}(V \cdot \text{ISV} \cdot U^T) \\
\end{align*}
\]

Filter

\[
\text{Filter}(t, x, \text{ord max}) := \begin{align*}
& \text{n} \leftarrow \text{last}(t) \\
& \quad \text{for } i \in 0..n \\
& \quad \quad \text{for } j \in 0..3 \\
& \quad \quad \quad A_{i,j} \leftarrow \left(t_i\right)^j \\
& \quad \text{X} \leftarrow \text{LeftInv}(A) \cdot x \\
& \quad \text{x 0.trend} \leftarrow A \cdot X \\
& \quad \text{x 0.red} \leftarrow x - \text{x 0.trend} \\
& \quad \Delta t \leftarrow t_n - t_0 \\
& \quad \Delta x \leftarrow \frac{x_0 \text{ red} - x_0 \text{ red}_0}{n} \\
& \quad \text{for } i \in 0..n \\
& \quad \quad x_{0.\text{ red}_i} \leftarrow x_{0.\text{ red}_i} - \frac{\Delta x \ 0.\text{ red}}{n} \\
& \quad \text{x 0.red.F} \leftarrow \text{cfft}(x \ 0.\text{ red}) \\
& \quad \text{for } k \in \text{ord max} + 1..n - \text{ord max}
\end{align*}
\]
\[
x 0,\text{red}.F_k \leftarrow 0
\]
\[
\omega \leftarrow \frac{2\pi}{\Delta t}
\]

for \( k \in 1.. \text{ord}_{\text{max}} \)

\[
x 1,\text{red}.F_k \leftarrow x 0,\text{red}.F_k \cdot (-k \cdot \omega \cdot i)
\]
\[
x 1,\text{red}.F_{n+1-k} \leftarrow x 0,\text{red}.F_{n+1-k} \cdot (k \cdot \omega \cdot i)
\]
\[
x 2,\text{red}.F_k \leftarrow x 0,\text{red}.F_k \cdot (-k \cdot \omega \cdot i)^2
\]
\[
x 2,\text{red}.F_{n+1-k} \leftarrow x 0,\text{red}.F_{n+1-k} \cdot (k \cdot \omega \cdot i)^2
\]

\[
x 0,\text{red} \leftarrow \text{Re} (\text{icfft} (x 0,\text{red}.F))
\]
\[
x 1,\text{red} \leftarrow \text{Re} (\text{icfft} (x 1,\text{red}.F))
\]
\[
x 2,\text{red} \leftarrow \text{Re} (\text{icfft} (x 2,\text{red}.F))
\]

for \( i \in 0.. n \)

\[
x 0_i \leftarrow x 0,\text{red}_i + i \cdot \frac{\Delta x 0,\text{red}}{n} + x 0,\text{trend}_i
\]
\[
x 1,\text{trend} \leftarrow \sum_{k=1}^{3} k \cdot X_k \cdot A^{<k-1>}
\]
\[
x 1 \leftarrow x 1,\text{red} + \frac{\Delta x 0,\text{red}}{\Delta t} + x 1,\text{trend}
\]
\[
x 2,\text{trend} \leftarrow \sum_{k=2}^{3} k! \cdot X_k \cdot A^{<k-2>}
\]
\[
x 2 \leftarrow x 2,\text{red} + x 2,\text{trend}
\]
\[
\begin{bmatrix}
x 0 \\
x 1 \\
x 2
\end{bmatrix}
\]
Schmiechen: Re-evaluation of quasisteady model propulsion tests with VWS Mod. 2491.0/1340

**Evaluation of model data VWS 2491/1340 according to rational method proposed**

**Test identification**  
TID := "VWS 2491/1340"

**Date of test**  
Date := 860909

**Test No.**  
Test := 8

**Basic data**

**Ship model VWS Mod. 2491.0**  
Barge Carrier, which has not been built, body plan and contours of stem and stern to found in the first appendix.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>6.5 m</td>
<td></td>
</tr>
<tr>
<td>Breadth</td>
<td>1.00 m</td>
<td></td>
</tr>
<tr>
<td>Draught</td>
<td>0.255 m</td>
<td></td>
</tr>
<tr>
<td>Displacement</td>
<td>1.431 m³</td>
<td></td>
</tr>
<tr>
<td>Block coefficient</td>
<td>0.8633</td>
<td></td>
</tr>
<tr>
<td>Density of tank water</td>
<td>1.00·10³ kg·m⁻³</td>
<td></td>
</tr>
<tr>
<td>Mass, model</td>
<td>1431.0000 kg</td>
<td></td>
</tr>
<tr>
<td>Mass, added</td>
<td>0.8679 kg</td>
<td></td>
</tr>
</tbody>
</table>

Thus the ship is much fuller than the equivalent half-ellipsoid and added mass data of ellipsoids provide only very crude estimates. The following value has been 'read' from figure 67 on pages 244-245 in the monograph of W.G. Price and R.E.D. Bishop: Probabilistic Theory of Ship Dynamics. London: Chapman and Hall, 1974.

\[
m_x := \frac{0.5}{58} = 0.0259
\]

\[
M_{hyd} := M_{nom}^m m_x
\]

\[
M_{hyd,S} := \rho \left(0.15 \cdot \pi \cdot B \cdot Tg^2\right)
\]

According to Sainsbury (Ship and Boat Builder 1963/12)
\[
m_{x,\text{nom}} := \frac{M_{\text{hyd}} S}{M_{\text{hyd}}} m_x
\]
\[
m_{x,\text{nom}} = 0.0214
\]

Model scale \[\lambda := 37.23\]
Location of trip wire \[x_{\text{wire}} := 19.25\]
Surface \[S := 8.967 \cdot m^2\]

**Propeller model VWS Prop. 1340**

CP propeller, right handed

Diameter of propeller \[D := 0.195 \cdot m\]
Disc area \[A_D := \frac{\pi}{4} D^2\]

Pitch ratio, design \[P_{D,\text{des}} := 0.825\]
Pitch ratio, actual \[P_{D,\text{act}} := 0.813\]
Number of blades \[Z := 4\]
Rate of revolutions at open water test \[n_{\text{open}} := 12 \cdot \text{Hz}\]

**Model test conditions**

Carriage velocity \[F_n := 0.168\]
\[v_{\text{carr}} := F_n \cdot \sqrt{g \cdot L}\]
\[v_{\text{carr}} = 1.3415\]

Frictional deduction \[C_F := 0.183\]
\[F_F := C_F \cdot \rho \cdot D^2 \cdot v_{\text{carr}}^2\]
\[F_F = 12.5234\]

**Input: Digitized .jpg files**

Data := READPRN("mod_data.dat")

nr := last(Data <0>)
ns := 0

time
\[t_r := \text{Data}_{n_s + r, 0 \cdot \text{sec}}\]
\[t := t \cdot \text{sec}^{-1}\]

rate of revolutions
\[n_{\text{raw}} := \text{Data}_{n_s + r, 1 \cdot \text{Hz}}\]
\[n_{\text{raw}} := n_{\text{raw}} \cdot \text{Hz}^{-1}\]

relative shift of model
\[s_{\text{raw}} := \text{Data}_{n_s + r, 4 \cdot m}\]
\[s_{\text{raw}} := s_{\text{raw}} \cdot m^{-1}\]

thrust
\[T_{\text{raw}} := \text{Data}_{n_s + r, 3 \cdot N}\]
\[T_{\text{raw}} := T_{\text{raw}} \cdot N^{-1}\]

torque
\[Q_{\text{raw}} := \text{Data}_{n_s + r, 2 \cdot \text{Nm}}\]
\[Q_{\text{raw}} := Q_{\text{raw}} \cdot \text{Nm}^{-1}\]

MS 25.04.2014 14:04h
Rate of revolution faired

\[ \text{ord}_{\text{max}} := 15 \]

\[ [n_{\text{fair}}, n_1, n_2] := \text{Filter}(t, n_{\text{raw}}, \text{ord}_{\text{max}}) \]

\[ E_n = n_{\text{raw}} - n_{\text{fair}} \quad e_n := \frac{E_n}{\text{mean}(n_{\text{fair}})} \]

This value has been chosen as 'optimal', closest to the steady conditions.

\[ \text{stdev}(E_n) = 0.0541 \]

\[ n_m := \text{mean}(n_{\text{raw}}) \]

\[ n_m = 9.8880 \]

Velocity and acceleration

\[ [s_{\text{rel}}, v_{\text{rel}}, a_{\text{rel}}] := \text{Filter}(t, s_{\text{raw}}, \text{ord}_{\text{max}}) \]

\[ E_s := s_{\text{raw}} - s_{\text{rel}} \quad \text{stdev}(E_s) = 0.0032 \]
Schmiechen: Re-evaluation of quasisteady model propulsion tests with VWS Mod. 2491.0/1340

Surge

\[ s_{\text{raw}} \times 10^{-3} \]

\[ s_{\text{rel}} \times 10^{-3} \]

Noise in mm

\[ E_{s} \times 10^{-3} \]

Relative speed

\[ v_{\text{rel}} \]

Acceleration

\[ a_{\text{rel}} \]
'Final' values

\[ v_{\text{fair}} = v_{\text{carr}} + v_{\text{rel}} \]
\[ a_{\text{fair}} = a_{\text{rel}} \]

Scrutinize data
Correlate torque and thrust

Torque/thrust correlation

Something has happened in the measurements of the higher torque values? Was there a problem with the dynamometer or did the flow pattern at the model propeller suddenly change?

The systematic problems above \( T = 32 \) N, \( Q = 0.8 \) Nm have been observed earlier and have already been mentioned explicitly in the basic VWS report No. 1100/87. There may have been many reasons for this behaviour, which has not been observed in the other runs. After much deliberation torque data are being corrected according to 'initial' linear correlation.

'Correct' torque values

\[
\text{Red} \left( T, Q, T_{\text{lim}} \right) = \begin{cases} 
  j &\leftarrow 0 \\
  k &\leftarrow 0 \\
  \text{for } i \in 0..\text{last}(T) \\
  T_{\text{red}} &\leftarrow T_i \text{ if } T_i < T_{\text{lim}} \\
  Q_{\text{red}} &\leftarrow Q_i \text{ if } T_i < T_{\text{lim}} \\
  j &\leftarrow j + 1 \text{ if } T_i < T_{\text{lim}} \\
  T_{\text{res}} &\leftarrow T_i \text{ if } T_i \geq T_{\text{lim}} \\
  Q_{\text{res}} &\leftarrow Q_i \text{ if } T_i \geq T_{\text{lim}} \\
  k &\leftarrow k + 1 \text{ if } T_i \geq T_{\text{lim}} \\
  \left[ T_{\text{red}} \ Q_{\text{red}} \ T_{\text{res}} \ Q_{\text{res}} \right] 
\end{cases}
\]
\[ T_{\text{lim}} = 32 \]

\[
\begin{bmatrix}
T_{\text{red}} & Q_{\text{red}} & T_{\text{res}} & Q_{\text{res}}
\end{bmatrix} := \text{Red}(T_{\text{raw}}, Q_{\text{raw}}, T_{\text{lim}})
\]

**Correlation of reduced sets**

\[ j := 0..\text{last}(T_{\text{red}}) \quad A_{\text{red},0} := 1 \quad A_{\text{red},1} := T_{\text{red}} \]

\[ X_{\text{red}} := \text{LeftInv}(A_{\text{red}}) \cdot Q_{\text{red}} \]

**'Correct' torque values**

\[ k := 0..\text{last}(T_{\text{res}}) \quad A_{\text{res},0} := 1 \quad A_{\text{res},1} := T_{\text{res}} \]

\[ Q_{\text{corr}} := A_{\text{res}} \cdot X_{\text{red}} \]

![Graph of torque values](image)

**'Correct' torque values replaced**

\[ \text{Rep}(T, Q, Q_{\text{corr}}, T_{\text{lim}}) := \begin{cases} 
  k & \text{if } T_i \geq T_{\text{lim}} \\
  k + 1 & \text{if } T_i < T_{\text{lim}} 
\end{cases} \]

\[ Q_{\text{corr}} := \text{Rep}(T_{\text{raw}}, Q_{\text{raw}}, Q_{\text{corr}}, T_{\text{lim}}) \]
Fair torque, thrust and force values

\[
A_{\text{fair},0} := \left( n_{\text{fair}} \right)^2 \\
X_T := \text{LeftInv}(A_{\text{fair}}) \cdot T_{\text{raw}} \\
X_Q := \text{LeftInv}(A_{\text{fair}}) \cdot Q_{\text{corr}} \\
T_{\text{fair}} := A_{\text{fair},0} \cdot X_T \\
Q_{\text{fair}} := A_{\text{fair},1} \cdot X_Q \\
E_T := T_{\text{raw}} - T_{\text{fair}} \\
E_Q := Q_{\text{corr}} - Q_{\text{fair}} \\
\text{stdev}(E_T) = 0.4704 \\
\text{stdev}(E_Q) = 0.0117 \\
\text{e}_T := \frac{E_T}{\text{mean}(T_{\text{raw}})} \\
\text{e}_Q := \frac{E_Q}{\text{mean}(Q_{\text{corr}})}
\]

Fair ed thrust and torque data
Normalize polynomial
\[ j := 0..2 \]
\[ X_{KTH}^j := \frac{\chi_T^j}{\rho \cdot D^4 - j} \quad \quad X_{KPH}^j := \frac{2 \pi \cdot \chi_Q^j}{\rho \cdot D^5 - j} \]

Thrust and power ratios as functions of hull advance ratio
\[ k_{TH}(jH) := \sum_j X_{KTH}^j j^H \quad \quad k_{PH}(jH) := \sum_j X_{KPH}^j j^H \]
Re-evaluation of quasisteady model propulsion tests with VWS Mod. 2491.0/1340

Recording of raw and faired values

\[
\begin{align*}
\text{Dat}_{\text{raw}}^{<0>} & : = t \\
\text{Dat}_{\text{raw}}^{<1>} & : = n_{\text{raw}} \\
\text{Dat}_{\text{raw}}^{<2>} & : = v_{\text{fair}} \\
\text{Dat}_{\text{raw}}^{<3>} & : = a_{\text{fair}} \\
\text{Dat}_{\text{raw}}^{<4>} & : = Q_{\text{raw}} \\
\end{align*}
\]

\text{WRITEPRN} ("dat_raw.dat") := \text{Dat}_{\text{raw}}

\[
\begin{align*}
\text{Dat}_{\text{fair}}^{<0>} & : = t \\
\text{Dat}_{\text{fair}}^{<1>} & : = n_{\text{fair}} \\
\text{Dat}_{\text{fair}}^{<2>} & : = v_{\text{fair}} \\
\text{Dat}_{\text{fair}}^{<3>} & : = a_{\text{fair}} \\
\text{Dat}_{\text{fair}}^{<4>} & : = Q_{\text{corr}} \\
\end{align*}
\]

\text{WRITEPRN} ("dat_fair.dat") := \text{Dat}_{\text{fair}}

Identify nominal wake fraction

Problem solved

As the detailed numerical exercises have shown the problem of the performance evaluation solely based on the results of quasi-steady propulsion tests is singular.

The only way to solve the problem is to provide an additional axiom or convention permitting to identify the nominal wake fraction, the phenomenological parameter in the wake axiom.

The additional axiom postulated before is that the hydraulic or pump efficiency of the propeller has a maximum at the centre of the range of interest.

In earlier evaluations this axiom has been applied without appropriate scrutiny to randomly available samples. The following procedure the 'range of interest' is changed until the postulate is met.

Explanation added

The axiom, a condition limiting the complexity of the model, has been adopted to get along with only two parameters to be identified in a robust procedure. Consequently this condition has to be provided for by appropriate selection of the range investigated.

After all the procedure is meeting the standards originally envisaged.

The detailed analysis reveals that the excellent results obtained earlier have been strictly accidental. The hydraulic efficiency happened to be stationary in the sample randomly selected!

According to the above explanation all attempts to identify the two parameters from randomly chosen propulsion data, may be at only two conditions, are doomed to fail 'by definition', due to the model purposely simplified.
**Determine range of data**

\[ J_{H.fair.r} = \frac{v_{fair}}{D \cdot n_{fair}} \]

\[ J_{H.fair.min} \leftarrow \min (J_{H.fair}) \quad J_{H.fair.min} = 0.6370 \]

\[ J_{H.fair.max} \leftarrow \max (J_{H.fair}) \quad J_{H.fair.max} = 0.7871 \]

**Determine jet efficiency**

*Based on axiom of jet efficiency and on thrust identity!*

\[ j_{H} \leftarrow J_{H.fair.mean} \]

\[ \omega_{TJ} : 0.5 \]

\[ h_{TJ} : 0.7 \]

*Given*

\[ \frac{2 \cdot k_{TH} (j_{H})}{\pi \cdot j_{H} \cdot (1 - \omega_{TJ} \cdot h_{TJ})} = 1 \cdot h_{TJ}^2 - h_{TJ} \]

\[ H_{TJ} (\omega_{TJ} \cdot j_{H}) \leftarrow \text{Find}(h_{TJ}) \]

\[ H_{TJ,T} (\omega_{TJ} \cdot j_{H}) \leftarrow \text{for } i \in 0.. \text{last}(j_{H}) \]

\[ \eta_{TJ,i} \leftarrow H_{TJ} (\omega_{TJ} \cdot j_{H,i}) \]

\[ \eta_{TJ} \]
Based on axiom of constant hydraulic efficiency!

\[ h_{TP} := 0.8 \]

Given

\[ h_{TJ} = \frac{h_{TP}}{\eta_{JP}} \left( 1 - \omega_{TJ} h_{TJ} \right) \]

\[ H_P(\omega_{TJ}, \eta_{JP}, h_{TP}) := \text{Find} \left( h_{TJ} \right) \]

\[ H_{TJ,P}(\omega_{TJ}, h_{JP,m}, h_{TPH}, j_H) := \begin{cases} \eta_{TJ} \leftarrow H_P(\omega_{TJ}, h_{JP,m}, h_{TPH_i}) \\ \eta_{TJ} \end{cases} \quad \text{for } i \in 0.. \text{last}(j_H) \]

Solve for nominal wake and mean hydraulic efficiency

\[ \omega_{TJ} := 0.57 \quad h_{JP,m} := 0.76 \]

Given

\[ H_{TJ,P}(\omega_{TJ}, h_{JP,m}, h_{TPH}, j_H) = H_{TJ,T}(\omega_{TJ}, j_H) \]

\[ \text{JetEff}(\omega_{TJ}, h_{JP,m}, h_{TPH}, j_H) := \text{MinErr}(\omega_{TJ}, h_{JP,m}) \]

Determine maximum hydraulic efficiency

\[ n := 5 \]
\[ \Delta j := 0.001 \]
\[ j_{H,c} := J_{H,fair.min} \]

\[ \text{Index}(v, v_m) := \begin{cases} j \leftarrow 0 \\ \text{while } v_j \neq v_m \\ j \leftarrow j + 1 \\ j \end{cases} \]
$\Delta J(j_{H,c}, \Delta j) := \begin{align*}
&| \begin{align*}
&j_{H_1} \leftarrow j_{H,c} + \Delta j(i-n) \\
&k T_i \leftarrow k T H(j_{H_1}) \\
&k P_i \leftarrow k P H(j_{H_1}) \\
&h T P H_i \leftarrow k P_i \\
&\Omega \leftarrow \text{JetEff}(\omega T J, h J P.m, h T P H, j H) \\
&\omega T J \leftarrow \Omega_0 \\
&h T J \leftarrow H T J.T(\omega T J, j H) \\
&| \begin{align*}
&\omega_i \leftarrow \omega T J \cdot h T J_i \\
&h J P_i \leftarrow h T P H_i \left(\frac{1 - \omega_i}{h T J_i}\right) \\
&h J P.m \leftarrow \max(h J P) \\
&m \leftarrow \text{Index}(h J P, h J P.m) \\
&\Delta j_{H} \leftarrow j_{H_m} - j_{H,c} \\
&\Delta j_H
\end{align*} | \end{align*}
\end{align*}$

$J_{H,c} := \text{root}(\Delta J(j_{H,c}, \Delta j), j_H)$

$J_{H,c} = 0.6984$

This result 'explains' why the former evaluation with the value 0.7 has been accidentally correct!
SampRange(\(j_{H,c}, \Delta j\)) :=

\[
\begin{align*}
  j_{H, i} &:= j_{H, c} + \Delta j \cdot (i - n) \\
  k_{T, i} &:= k_{TH}(j_{H, i}) \\
  k_{P, i} &:= k_{PH}(j_{H, i}) \\
  h_{TPH, i} &:= \frac{k_{T, i} \cdot j_{H, i}}{k_{P, i}}
\end{align*}
\]

\[\Omega \leftarrow \text{JetEff}(\omega_{TJ}, h_{JP.m}, h_{TPH}, j_H)\]

\[
\begin{bmatrix}
  j_H \\
  k_T \\
  k_P \\
  h_{TPH} \\
  \Omega
\end{bmatrix}
\]

\[S := \text{SampRange}(J_{H,c}, \Delta j)\]

\[w_{TJ} := (S_4)_0\]

\[\eta_{JP.m} := (S_4)_1\]

Evaluate over a wide range

\[J_{H,c} := \frac{\text{round}(10 \cdot J_{H,\text{fair.mean}})}{10}\]

\[J_{H,c} = 0.7000\]

\[\Delta j := \frac{\text{round}[10 \cdot (J_{H,\text{fair.max}} - J_{H,\text{fair.min}})]}{10 \cdot n}\]

\[\Delta j = 0.0400\]
Determine derived magnitudes

\[
\begin{aligned}
J_H \\
K_T \\
K_P \\
\eta_TPH \\
\Omega \\
\end{aligned}
:= \text{SampRange}(J_{H,c}, \Delta j)
\]

\[
\begin{aligned}
i &:= 0 \ldots \text{last}(J_H) \\
\eta_{TJ} &:= H \cdot T_{J,T} \cdot (w_{TJ} \cdot J_H) \\
w_i &:= w_{TJ} \cdot \eta_{TJ_i} \\
J_{P_i} &:= J_{H_i} \cdot (1 - w_i) \\
\end{aligned}
\]

\[
\begin{aligned}
\eta_{TP_i} &:= \frac{K_T \cdot J_{P_i}}{K_{P_i}} \\
\eta_{JP_i} &:= \frac{\eta_{TP_i}}{\eta_{TJ_i}} \\
c_T &:= \frac{8 \cdot K_{T_i}}{\pi \cdot (J_{P_i})^2}
\end{aligned}
\]

'Equivalent' open water chart of CP propeller model in the behind condition according to rational procedure proposed.
Compare with traditional evaluation based on propeller open water test results

Data

\[
\begin{bmatrix}
0.35 & 48.0 & 63.5 \\
0.40 & 43.0 & 59.5 \\
0.45 & 38.0 & 53.0 \\
0.50 & 33.0 & 48.0 \\
0.55 & 28.0 & 43.0 \\
0.60 & 23.0 & 37.5 \\
0.65 & 17.5 & 32.0 \\
\end{bmatrix}
\]

KT and 10 KQ values read in mm from Fig. 0.2 in VWS Bericht Nr. 1126/88

scale := 200

\[
\begin{align*}
J_{\text{P.open}} & := \text{Data\ prop} <0> \\
K_{T,\text{raw}} & := \frac{\text{Data\ prop} <1> \cdot \text{scale}}{10} \\
K_{P,\text{raw}} & := \frac{2 \cdot \pi \cdot \text{Data\ prop} <2> \cdot \text{scale}}{10} \\
k & := 0.. \text{last}\{J_{\text{P.open}}\} \\
A_{\text{JP.open}}_{k,j} & := \{J_{\text{P.open}}_{k}\}^j \\
X_{\text{KT.open}} & := \text{LeftInv}\left(A_{\text{JP.open}}\right) \cdot K_{T,\text{raw}} \\
X_{\text{KPo}} & := \text{LeftInv}\left(A_{\text{JP.open}}\right) \cdot K_{P,\text{raw}} \\
K_{T,\text{P}} & := A_{\text{JP.open}} \cdot X_{\text{KT.open}} \\
K_{P,\text{P}} & := A_{\text{JP.open}} \cdot X_{\text{KPo}}
\end{align*}
\]

Thrust and power ratios as functions of propeller open water advance ratio

\[
\begin{align*}
k_{T,\text{open}}\left(j_{\text{p}}\right) & := \sum_j X_{\text{KT.open}}^{j_{\text{p}}} j_{\text{p}}^j \\
k_{P,\text{open}}\left(j_{\text{p}}\right) & := \sum_j X_{\text{KPo}}^{j_{\text{p}}} j_{\text{p}}^j \\
K_{T,\text{open}} & := k_{T,\text{open}}\left(J_{\text{p}}\right) \\
K_{P,\text{open}} & := k_{P,\text{open}}\left(J_{\text{p}}\right)
\end{align*}
\]
Compare with open water values

**Thrust ratios**

- $K_T$
- $K_{T,\text{open}}$
- $K_{T,\text{open}}$
- $K_{T,\text{open}}$
- $K_{T,\text{open}}$
- $K_{T,\text{open}}$
- $K_{T,\text{open}}$

**Power ratios**

- $K_P$
- $K_{P,\text{open}}$
- $K_{P,\text{open}}$
- $K_{P,\text{open}}$
- $K_{P,\text{open}}$
- $K_{P,\text{open}}$
- $K_{P,\text{open}}$

Wake fractions based the model propeller open water performance

**Thrust identity**

$j_{PT} := 1$

Given

$k_{T,\text{open}}(j_{PT}) = k_{TH}$

$1 \cdot PT(k_{TH}) := \text{Find}(j_{PT})$

$J_{PT_i} := 1 \cdot PT(K_{T_i})$

$w_{T_i} := 1 - \frac{J_{PT_i}}{J_{H_i}}$

$w_{\text{trad}} := w_{T}$
\[ j := 0 \ldots 1 \]

\[ A_{JH_{i,j}} := \langle J_{H_i} \rangle^j \]

\[ X_{WT} := \text{LeftInv}(A_{JH})^w T \]

\[ k_{WT}(jJ_{H}) := \sum_j X_{WT}^j H^j \]

**Power identity**

\[ j_{PP} := 1 \]

Given

\[ k_{P.open}(jPP) = \kappa PH \]

\[ 1^{PP}(\kappa PH) := \text{Find}(jPP) \]

\[ J_{PP_i} := 1^{PP}(K_{P_i}) \]

\[ w_{P} := 1 - \frac{J_{PP_i}}{J_{H_i}} \]

\[ j := 0 \ldots 1 \]

\[ A_{JH_{i,j}} := \langle J_{H_i} \rangle^j \]

\[ X_{WT} := \text{LeftInv}(A_{JH})^w T \]

\[ k_{WT}(jH) := \sum_j X_{WT}^j H^j \]
Determine propeller efficiencies: open condition

\[ \eta_{\text{TP.open}} = \frac{K_{\text{T.open}} J P_i}{K_{\text{P.open}}} \]

\[ c_{\text{T.open}} = \frac{8 K_{\text{T.open}}}{\pi (J P_i)^2} \]

\[ \eta_{\text{TJ.open}} = \frac{2}{1 + \sqrt{1 + c_{\text{T.open}}}} \]

\[ \eta_{\text{JP.open}} = \frac{\eta_{\text{TP.open}}}{\eta_{\text{TJ.open}}} \]
Determine resistance and thrust deduction fraction

Problem solved

As has been observed earlier the thrust deduction axiom in accordance with the global approximation of the thrust deduction theorem is too crude to permit the identification of reasonable energy wake fractions.

Accordingly further attempts have been made to replace that axiom but without success. By the way it has been noticed that the value of the longitudinal hydrodynamic inertia is crucially affecting the momentum balance and the final results.

Further it has been observed that the maximum order of the filter selected has considerable impact on the inertia identified. Accordingly a procedure has been developed to extrapolate from quasi-steady to steady conditions.

Determine time range

\[ t_m := \text{mean}(t) \]
\[ t_m = 66.5759 \]
\[ \Delta t_r := t - t_m \]

Determine velocity range

\[ v_m := \text{mean}(v_{\text{fair}}) \]
\[ v_m = 1.3417 \]
\[ \Delta v_{\text{fair}_r} := v_{\text{fair}_r} - v_m \]
\[ \text{min}(v_{\text{fair}}) = 1.3118 \]
\[ \text{max}(v_{\text{fair}}) = 1.3621 \]

Determine thrust deduction fraction

based on simple axiom in accordance with global approximation of thrust deduction theorem

\[ J_{H,\text{fair}_r} := \frac{v_{\text{fair}_r}}{D \cdot n_{\text{fair}_r}} \]
\[ \eta_{TJ,\text{fair}_r} := H_{TJ} \cdot \frac{w_{\text{fair}_r}}{J_{H,\text{fair}_r}} \]
\[ w_{\text{fair}_r} := \frac{w_{\text{fair}_r}}{TJ} \cdot \eta_{TJ,\text{fair}_r} \]
\[ F_{\text{fair}_R_r} := F_{TJ} \left( 1 - \frac{a_{\text{fair}_r}}{g} \right) - M_{\text{nom}} \left( 1 + m_{\text{x,nom}} \right) a_{\text{fair}_r} \]
\[ A_{MR_{r,0}} := \eta_{TJ,\text{fair}_r} \cdot T_{\text{fair}_r} \]
\[ k := 0..1 \]
\[ A_{MR_{r,k+1}} := (\Delta v_{\text{fair}_r})^k \]
\[ A_{MR_{r,0}} := \Delta t_r \]
\[ B_{MR_{r}} := T_{\text{fair}_r} + F_{\text{fair}R_{r}} \]
\[ X_{MR} := \text{LeftInv}(A_{MR}) \cdot B_{MR} \]
\[ E_{MR} := B_{MR} - A_{MR} \cdot X_{MR} \]
\[ \begin{bmatrix} 0.399 \\ 33.715 \\ 74.445 \\ -0.016 \end{bmatrix} \]
\[ |E_{MR}| = 0.0272 \]
\[ B_{MR} \]
\[ M_{\text{hyd.id}} := \frac{E_{MR} \cdot a_{\text{fair}}}{a_{\text{fair}} \cdot a_{\text{fair}}} \]
\[ M_{\text{hyd.id}} = -129.6873 \]

\[ t_{\text{TJ}} := X_{MR_0} \]
\[ \text{thd} := t_{\text{TJ}} \eta_{\text{TJ}} \]
\[ R_r := \sum_k (\Delta v_{\text{fair}_r})^k \cdot X_{MR_{k+1}} \]

**Determine total inertia**

\[ F_{\text{fair}I_{r}} := F \cdot \left( 1 - \frac{a_{\text{fair}_r}}{g} \right) - R_r \]
\[ A_{MI_{r,0}} := \eta_{\text{TJ}.\text{fair}} \cdot T_{\text{fair}_r} \]
\[ A_{MI_{r,1}} := a_{\text{fair}_r} \]
\[ A_{MI_{r,2}} := \Delta t_r \]
\[ B_{MI_{r}} := T_{\text{fair}_r} + F_{\text{fair}I_{r}} \]
\[
X_{MI} := \text{LeftInv}(A_{MI}) \cdot B_{MI} \\
E_{MI} := B_{MI} - A_{MI} \cdot X_{MI}
\]

\[
X_{MI} = \begin{bmatrix} 0.4015 \\ 1329.2432 \\ -0.0158 \end{bmatrix}
\]

\[
E_{MI} = -4, 2, 0, -2, -4
\]

\[
E_{MR} = 4, 2, 0, 2, 4
\]

\[
t_{TJ} := X_{MI0}
\]

\[
\text{thd} := t_{TJ} \cdot \eta_{TJ}
\]

\[
\Delta M = M_{\text{nom}} \cdot (1 + m_{x,\text{nom}}) - X_{MI1}
\]

\[
\Delta M = 132.3991
\]

\[
m_{x,\text{meas}} := \frac{X_{MI1}}{M_{\text{nom}}} - 1
\]

\[
m_{x,\text{meas}} = -0.0711
\]

**Extrapolation from quasi-steady to steady conditions**

\[
\begin{bmatrix}
16 & 1300.70 & -0.091 \\
12 & 1376.69 & -0.03795 \\
10 & 1385.36 & -0.03189 \\
8 & 1393.59 & -0.02614 \\
7 & 1423.06 & -0.00555 \\
6 & 1432.24 & 0.00087 \\
5 & 1437.10 & 0.00426 \\
4 & 1435.18 & 0.00292
\end{bmatrix}
\]

\[
\text{ord \_max} = \text{inertia}^{<0>}, \quad M_{\text{tot,meas}} := \text{inertia}^{<1>}, \quad m_{x,\text{meas}} := \text{inertia}^{<2>}
\]

k.max, M.tot.meas, m.x.meas determined by repeated computations with varying maximum order of the filter.
Schmiechen: Re-evaluation of quasisteady model propulsion tests with VWS Mod. 2491.0/1340

\[
\begin{align*}
  j_{\text{max}} & := \text{last}(\operatorname{ord}_{\text{max}}) \\
  j & := 0 \ldots j_{\text{max}} \\
  A_{O,j,0} & := 1 \\
  A_{O,j,1} & := (\operatorname{ord}_{\text{max}})^2 \\
  X_M & := \text{LeftInv}(A_O) \cdot M_{\text{tot.meas}} \\
  M_{\text{tot.steady}} & := X_M 0 \\
  M_{\text{tot.steady}} & = 1446.3679
\end{align*}
\]

Plot of extrapolation

\[
\begin{align*}
  \text{inert}(\operatorname{ord}) & := X_{M_0} + X_{M_1} \cdot \operatorname{ord}^2 \\
  jj & := 0 \ldots 16 \\
  \text{ord}_{jj} & := jj \\
  M_{\text{tot.fair},jj} & := \text{inert}(\operatorname{ord}_{jj})
\end{align*}
\]

Scrutinise result

\[
M_{\text{steady}} := \frac{M_{\text{tot.steady}}}{1 + m_x}
\]

\[
M_{\text{steady}} = 1409.9049 \\
M_{\text{nom}} = 1431.0000
\]

Difference in 'observed' and nominal model mass

\[
\Delta M := M_{\text{steady}} - M_{\text{nom}} \\
\Delta M = -21.0951
\]

Of course this result is strictly accidental. But it may also be speculated that the model was not fully ballasted, two 10 kg 'weight pieces' missing for whatever reason. In view of the uncertainty there is no chance to identify the coefficient of the hydrodynamic inertia.
'Ship efficiencies'

\[ \eta_{RT_i} := \frac{1 - \text{thd}_i}{1 - w_i} \]

Hull efficiency, 'Rumpfeinflussgrad'

\[ \eta_{RJ_i} := \eta_{RT_i} \cdot \eta_{TJ_i} \]

Configuration efficiency, 'Konfigurationsgütegrad'

\[ \eta_{RP_i} := \eta_{RJ_i} \cdot \eta_{JP_i} \]

Propulsive efficiency, 'Gesamtgütegrad'

\[ \eta_{\text{rot}_i} := 1 \]

Rotative efficiency, equals 1 by definition in the rational theory!

Ship efficiencies, rational

Wake fractions

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**Compare with traditional evaluation based on hull towing test**

**Resistance, traditional: hull towing**

**Scrutiny of data**

$$\begin{pmatrix}
0.90 & 13.6 \\
1.00 & 16.8 \\
1.10 & 20.7 \\
1.20 & 25.2 \\
1.30 & 30.4 \\
1.35 & 33.2 \\
\end{pmatrix}$$

Values \(v\) in m/s, of \(R\) in N
read from Fig. 3.4 in
VWS Bericht Nr. 1126/88.
They conicide with those in
VWS Report No. 1100/87.

\(v_{\text{tow}} := \text{Data}_{\text{tow}}^{<0>} \cdot \text{m-sec}^{-1}\)

\(R_{\text{tow}} := \text{Data}_{\text{tow}}^{<1>} \cdot \text{N}\)

**Fair data**

\(j := 0..\text{last}(v_{\text{tow}})\)

\(k := 0..3\)

\[X_{R,\text{trad}} := \text{LeftInv}(A_{R,\text{trad}}) \cdot R_{\text{tow}}\]

\(v_{\text{plt}} := 1.31 + j \cdot 0.01\)

\[A_{R,\text{plt},j,k} := (v_{\text{plt},j})^k\]

\(R_{\text{trad,plt}} := A_{R,\text{plt}} \cdot X_{R,\text{trad}}\)

**Resistance, rational**

\(j := 0..\text{last}(v_{\text{fair}})\)

\(k := 0..3\)

\[X_{R,\text{rat}} := \text{LeftInv}(A_{R,\text{rat}}) \cdot R\]

\(R_{\text{rat,plt}} := A_{R,\text{plt}} \cdot X_{R,\text{rat}}\)
Schmiechen: Re-evaluation of quasisteady model propulsion tests with VWS Mod. 2491.0/1340

\[ A_{R,tow,r,k} := \left( v_{fair,r} \right)^k \]

\[ R_{tow} := A_{R,tow,r} \times R_{trad} \]
Thrust deduction fraction, traditional

\[ k := 0 .. 1 \]
\[ A_{\text{thd},k} := (J H_{\text{fair}})^k \cdot T_{\text{fair}} \]
\[ B_{\text{thd}} := T_{\text{fair}} + F_{\text{fairR}} - R_{\text{tow}} \]
\[ X_{\text{thd}} := \text{LeftInv}(A_{\text{thd}}) \cdot B_{\text{thd}} \]
\[ X_{\text{thd}} = \begin{bmatrix} 0.1200 \\ 0.2612 \end{bmatrix} \]
\[ \text{thd}_{\text{trad},i} := \sum_k (J H_i)^k \cdot X_{\text{thd},k} \]

Thrust deduction fractions

'Ship efficiencies', traditional

\[ \eta_{\text{RP.trad},i} := (1 - \text{thd}_{\text{trad},i}) \cdot \frac{K T_i \cdot J H_i}{K P_i} \]
Propulsive efficiency, 'Gesamtgütegrad'

\[ \eta_{\text{RT.trad},i} := \frac{1 - \text{thd}_{\text{trad},i}}{1 - \text{w}_{\text{trad},i}} \]
Hull efficiency, 'Rumpfeinflussgrad'

\[ \eta_{\text{TP.trad},i} := \frac{\eta_{\text{RP.trad},i}}{\eta_{\text{RT.trad},i}} \]
Behind efficiency

\[ \eta_{\text{rot.trad},i} := \frac{\eta_{\text{TP.trad},i}}{\eta_{\text{TP.open},i}} \]
Rotative efficiency, Anordnungsgütegrad

\[ \eta_{\text{RJ.trad},i} := \eta_{\text{RT.trad},i} \cdot \eta_{\text{TJ.open},i} \]
Configuration efficiency, 'Konfigurationsgütegrad'
Compare with results of rational evaluation

Propulsive efficiencies

Hull efficiencies

Behind efficiencies
Output of results for comparison with the results of quasi-steady 'model' trial (mod_trial.mcd)

\[
\text{res\_mod\_eval} := \begin{bmatrix}
\nu_{\text{plt}} & R_{\text{rat.plt}} & R_{\text{trad.plt}} \\
J_H & \eta_R & \eta_{\text{RP.trad}}
\end{bmatrix}
\]

WRITEPRN("Res_mod_eval") := res_mod_eval
Some conclusions

This rigorous re-evaluation of the model test has confirmed the results of the former re-evaluation and shown why that evaluation accidentally happened to be correct concerning the determination of the nominal wake fraction etc.

Concerning the determination of the resistance and thrust deduction fraction numerical studies have shown that the momentum balance is crucially affected by the value of the hydrodynamic inertia assumed and thus the final values of the resistance and the thrust deduction fraction.

Further the analysis has shown that the values of the inertia identified strongly depend on the maximum order of the filter applied to the raw data. Accordingly a procedure has been developed to extrapolate from quasi-steady conditions to the steady condition.

In view of the remaining uncertainties the small value of the hydrodynamic inertia cannot be identified. A nominal value has been assumed according to Sainsbury.

Concerning the determination of the energy wake fraction the problems observed earlier have not yet been resolved, maybe they cannot be resolved in the context developed so far.

For the time being further analysis has to be delayed.

(The file had to be reprinted due to problems with the pdf-writer. MS 090626)

END
Model data VWS 2491/1340 re-evaluated